

# Forest owners' collective action against the risk of forest fire: a game theoretical approach<sup>1</sup>

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## ABSTRACT

This paper is a follow up on a earlier one (Mendes, 1998) where I proposed a series of models for forest owners associations represented as organisation made up of two groups of strategically interacting players: the forest owners who are members of the association and the board of directors they have elected. The directors decide on the amount of services provided by the association which can be public goods (collective representation of the members, promotion of their common interests, diffusion of general information about forest programmes and best forest management practices, etc.) and private goods and services (silvicultural works preventive of forest fires, technical advice, etc.). The models were set up as games in strategic form with complete information and no payoff uncertainty.

Here I pick up the second of, what is called in that previous paper, the "Portuguese" models and extend it in the following directions:

- there is payoff risk for the forest owners due to exogenous hazards (forest fires or others);
- forest owners can buy private services from the owners which contribute to reduce the losses resulting from those hazards.

The main focus in this paper is to derive the comparative static results about the demand of these private services by the forest owners.

**KEYWORDS:** forest owners' associations, public and private goods joint supply, game theory

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## 1. INTRODUCTION

Mendes (1998) proposed a series of models for forest owners associations represented as organisation made up of two groups of strategically interacting players: the forest owners who are members of the association and the board of directors they have elected. The directors decide on the amount of services provided by the association which can be public goods (collective representation of the members, promotion of their common interests, diffusion of general information about forest programmes and best forest management practices, etc.) and private goods and services (silvicultural works preventive of forest fires, technical advice, etc.).

"Being non profit organisations, there are no monetary rewards for the directors. So their motivation is assumed to come from the positive utility (good reputation, personal satisfaction) they get from the amount of services provided by the association.

Directors and forest owners are assumed to have other activities besides managing the association and their forest lands. So in allocating their time they have to take into account the costs and benefits (monetary and non monetary) they get from these different activities. In taking these time allocation decisions members and directors are strategically interdependent in the following sense:

- the benefits of the time devoted to forest management by the owners depend on the services they get from the association whose level is decided by the board of directors;
- depending on the regime of membership contributions, the level of services set by the directors might also have to take into account the owners' forest management decisions." (Mendes, 1998).

Given the fact that this strategic interdependence depends on the regime of membership contributions and on the type of services provided by the association, three different situations were examined in that paper. The first one called the "Portuguese" type of association is one where members contribute with fixed fees to the costs of the organisation. The second model is a development of this first "Portuguese" model where besides producing public goods, the associations also provide private services to the members for which they charge a price. The third model called the "Scandinavian" type of associations is one where the members contribute with a share of the gross value of their timber sales.

The models were set up as games in strategic form with complete information and no payoff uncertainty.

Here we will pick up the second "Portuguese" model from that paper and extend it in the following directions:

- there is payoff risk for the forest owners due to exogenous hazards (forest fires or others);
- forest owners can buy private services from the owners which contribute to reduce the losses resulting from those hazards.

The main focus in this paper is to derive the comparative static results about the demand of these private services by the forest owners.

## 2. ASSUMPTIONS

### Assumption 1

The output of the association consists in a public good produced in a quantity  $Q$  and a quantity  $R$  of private services.

### Assumption 2

In the internal organisation of the associations there are two types of players: the members and the board of directors they have elected for some fixed term.

### Assumption 3

The directors are considered as a single player and the fact that they are forest owners is ignored.

### Assumption 4

For the time frame of this model, the number of registered members is fixed.

### Assumption 5

Directors and members behave non co-operatively.

### Assumption 6

The directors don't get any monetary reward from their work for the association. Their incentive for this job comes from the fact that their utility  $W$  depends positively on the amount of services provided by the association to their members. Utility also depends positively on the following variables:

- $Y$ , the amount of consumer goods and services used for their personal consumption;
- $X$ , leisure.

### Assumption 7

Each forest owner faces an exogenous risk of loss in his forest production, the probability of loss being  $\pi_i$ . In this case he gets a fraction  $\alpha(R_i)$  of his forest production  $F_i(Z_i, Q, B_i)$  where:

- $F_i$  is a forest production function with positive first-order partial derivatives and negative second-order own-partial derivatives;
- $R_i$  is the amount of forest private services bought by the forest owners from the association;
- $B_i$  denotes other inputs of forest production supposed to be fixed.

### Assumption 8

Each forest owner behaves as an expected utility maximizer, his elementary utility  $U_i$  depending on the following variables:

- $Y_i$ , the amount of consumer goods and services used for their personal consumption;
- $X_i$ , leisure.

### Assumption 9

The behaviour of each member has to meet two economic constraints:

- a personal budget constraint;
- a time constraint.

#### **Assumption 10**

Each member's budget constraint is the following:

$$(1) \quad pY_i + m \leq \omega H_i + V_i + \alpha(R_i)F_i(Z_i, Q, B_i) \text{ in case an exogenous hazard happens}$$

or

$$pY_i + m + rR_i \leq \omega H_i + V_i + F_i(Z_i, Q, B_i) \text{ in the other case}$$

where  $m$  is the annual membership fee assumed to be the same for all the members and  $r$  is the price of the private services provided by the association.

#### **Assumption 11**

The directors set the price  $r$  of the private services and let the demand determine the quantity  $R$ .

#### **Assumption 12**

Each members' time constraint is the following:

$$(2) \quad T = X_i + H_i + Z_i \quad \text{where:}$$

- $X_i$  is leisure;
- $H_i$  is paid labour time;
- $Z_i$  is time devoted to forest management.

#### **Assumption 13**

The utility of the directors depends positively on  $Q$  and  $R$ .

#### **Assumption 14**

The behaviour of the directors has to meet three economic constraints:

- their personal budget constraint;
- the association's budget constraint;
- a time constraint.

#### **Assumption 15**

The directors' personal budget constraint is the following:

$$(3) \quad pY \leq \omega H + V \quad \text{where:}$$

- $p$  is the price of the consumer goods and services;
- $\omega$  is the income per unit of paid labour time;
- $H$  is paid labour time;
- $V$  is the non labour income.

#### **Assumption 16**

The directors' time constraint is the following:

$$(4) \quad T = X + H + Z \quad \text{where:}$$

- $T$  is the total time available;
- $X$  is leisure;

-  $Z$  is time devoted to the association.

### Assumption 17

The association's budget constraint takes the following form:

$$(5) \quad C[Q, R(r)] \leq n(Z)m + S(Z) + rR(r)$$

with the following properties:

$$(6) \quad \frac{\partial n}{\partial Z} > 0, \quad \frac{\partial S}{\partial Z} > 0 \quad \text{where:}$$

- $C$  is the total production cost of the public good;
- $n$  is the number of members who actually pay the membership fees;
- $S$  represents other sources of funding besides membership fees, namely public grants and other financial support obtained by the fund raising effort of the directors.

### 3. THE DIRECTORS' EQUILIBRIUM STRATEGIES

The directors' decision problem is the following:

$$(7) \quad \begin{aligned} & \underset{X, Y, Q, Z, r}{\text{Max}} \quad W(X, Y, Q, R) \\ & \text{s.t.} \quad pY \leq \omega H + V \\ & \quad T = X + H + Z \\ & \quad C(Q) \leq n(Z)m + S(Z) + rR(r) \quad X, Y, Z, Q \geq 0 \end{aligned}$$

The Kuhn-Tucker conditions for this problem are the following where  $\lambda$  and  $\beta$  are the Lagrange multipliers:

$$(8) \quad \frac{\partial \mathcal{L}}{\partial X} = \frac{\partial W}{\partial X} - \lambda \omega \leq 0, \quad X \geq 0, \quad X \frac{\partial \mathcal{L}}{\partial X} = 0$$

$$(9) \quad \frac{\partial \mathcal{L}}{\partial Y} = \frac{\partial W}{\partial Y} - \lambda p \leq 0, \quad Y \geq 0, \quad Y \frac{\partial \mathcal{L}}{\partial Y} = 0$$

$$(10) \quad \frac{\partial \mathcal{L}}{\partial Q} = \frac{\partial W}{\partial Q} - \beta \frac{\partial C}{\partial Q} \leq 0, \quad Q \geq 0, \quad Q \frac{\partial \mathcal{L}}{\partial Q} = 0$$

$$(11) \quad \frac{\partial \mathcal{L}}{\partial Z} = -\alpha \omega + \beta m \frac{\partial n}{\partial Z} + \beta \frac{\partial S}{\partial Z} \leq 0, \quad Z \geq 0, \quad Z \frac{\partial \mathcal{L}}{\partial Z} = 0$$

$$(12) \quad \frac{\partial \mathcal{L}}{\partial r} = \frac{\partial W}{\partial R} \frac{\partial R}{\partial r} + \beta \left[ R(r) + r \frac{\partial R}{\partial r} - \frac{\partial C}{\partial R} \frac{\partial R}{\partial r} \right] \leq 0, \quad r \geq 0, \quad r \frac{\partial \mathcal{L}}{\partial r} = 0$$

For an interior solution, from these conditions we get:

$$(13) \quad MRS_{QX} = \frac{C'}{mn' + S'}$$

Since the association provides both public and private goods the marginal cost  $C'$  depends on the quantity of these private services demanded by the members. So the directors' equilibrium strategies **loose the dominant strategies' feature** they would have if only public goods were provided to the members.

Looking now at the private services' price policy set by the directors we have to work with condition (12) to get the following result in the case of an interior solution:

$$(14) \quad \begin{aligned} \beta \left[ R(r) + r \frac{\partial R}{\partial r} - \frac{\partial C}{\partial R} \frac{\partial R}{\partial r} \right] &= - \frac{\partial W}{\partial R} \frac{\partial R}{\partial r} \Rightarrow \beta \left[ \frac{R(r)}{\frac{\partial R}{\partial r}} + r - \frac{\partial C}{\partial R} \right] = - \frac{\partial W}{\partial R} \Rightarrow \\ \Rightarrow \beta \left\{ r \left[ \frac{\frac{R(r)}{r}}{\frac{\partial R}{\partial r}} + 1 \right] - \frac{\partial C}{\partial R} \right\} &= - \frac{\partial W}{\partial R} \Rightarrow \beta \left[ r \left( 1 + \frac{1}{\varepsilon_r} \right) - \frac{\partial C}{\partial R} \right] = - \frac{\partial W}{\partial R} \end{aligned}$$

where  $\varepsilon_r$  is the price elasticity of the demand for private services.

Dividing this condition by the one for the public good we get:

$$(15) \quad \frac{r \left( 1 + \frac{1}{\varepsilon_r} \right) - \frac{\partial C}{\partial R}}{- \frac{\partial C}{\partial Q}} = \frac{\frac{\partial W}{\partial R}}{\frac{\partial W}{\partial Q}} \Leftrightarrow \frac{B_R}{B_Q} = MRS_{RQ}$$

where  $B_i$  is the marginal benefit of service  $i$ . What this result shows is that the optimal price for the private services is the one for which the marginal rate at which the directors are willing to substitute the private for the public goods is equal to the ratio of the marginal benefits of these services.

#### 4 THE MEMBERS' EQUILIBRIUM STRATEGIES

Each member faces the following decision problem:

$$(16) \quad \begin{aligned} \underset{X_i, Y_i, Z_i, R_i}{Max} \quad EU_i &= \pi U_i \left[ \frac{\omega}{p} (T - X_i - Z_i) + \frac{\alpha(R_i)}{p} F_i(Z_i, Q, B_i) - \frac{m}{p} - \frac{w}{p} R_i; X_i \right] + \\ &+ (1 - \pi) U_i \left[ \frac{\omega}{p} (T - X_i - Z_i) + \frac{\alpha(R_i)}{p} F_i(Z_i, Q, B_i) - \frac{m}{p} - \frac{w}{p} R_i; X_i \right] \\ s.t. \quad &X_i, Y_i, Z_i, R_i \geq 0 \end{aligned}$$

The first order conditions for this problem after simplifications are the following:

$$(17) \quad \pi \frac{\partial \alpha}{\partial R_i} F_i(Z_i, Q, B_i) = w$$

$$(18) \quad \{1 - [1 - \alpha(R_i)\pi]\} \frac{\partial F}{\partial Z_i} = \omega$$

$$(19) \quad \frac{\frac{\partial U_i}{\partial X_i}}{\frac{\partial U_i}{\partial Y_i}} = \frac{\omega}{p}$$

The first two equations uniquely determine the optimal values of the production endogenous variables:  $Z_i$  and  $R_i$ . So as usual in consumer-producer models where prices are exogenous to the decision maker, there is **separability** between the production decisions and the consumption and labour supply decisions. Besides separability, there is also **recursivity** in the sense that the production decisions determine the consumption and labour supply decisions but not vice-versa. The first two equations above characterize the production decisions. The third one characterizes the labour supply and the consumption decisions by the usual condition of equality between the marginal rate of substitution between leisure and the consumer goods and the real wage rate.

To get the sign of the partial derivatives of the forest owner's demand for private services from the association contributing to reduce the loss due to exogenous hazards we have to differentiate the system of the two equations (17) e (18) with respect to the variables which are exogenous to the forest owner. Here we will focus on the influence of the price of the private services supplied by the association and set by the board of directors. We get the following system:

$$(20) \quad \begin{cases} \pi \frac{\partial \alpha^2}{\partial^2 R_i} F_i(\cdot) \frac{\partial R_i}{\partial w} + \pi \frac{\partial \alpha}{\partial R_i} \frac{\partial F_i}{\partial Z_i} \frac{\partial Z_i}{\partial w} = 1 \\ \pi \frac{\partial \alpha}{\partial R_i} \frac{\partial F_i}{\partial Z_i} \frac{\partial R_i}{\partial w} + \{1 - [1 - \alpha(R_i)\pi]\} \frac{\partial F_i}{\partial Z_i} \frac{\partial Z_i}{\partial w} = 0 \end{cases}$$

Solving for  $\frac{\partial R_i}{\partial w}$  we get:

$$(21) \quad \frac{\partial R_i}{\partial w} = \frac{\{1 - [1 - \alpha(R_i)\pi]\} \frac{\partial F_i}{\partial Z_i}}{\Delta} < 0$$

where  $\Delta$  is the determinant of the system considered to be negative under the assumption of **decreasing marginal returns to the private services** provided by the association:

$$(22) \quad \frac{\partial \alpha^2}{\partial^2 R_i} < 0$$

From the system above we also get the value for the partial derivative of the labour supplied his forest farm by the forest owner with respect to the price of the private services supplied by the association:

$$(23) \frac{\partial Z_i}{\partial w} = \frac{-\pi \frac{\partial \alpha}{\partial R_i} \frac{\partial F_i}{\partial Z_i}}{\Delta} > 0$$

So under the assumption of this model raising the price of the private services supplied by the association would lower the demand of those services b the forest owners and raise the amount of time they supply to their forest farm.

## 5. REFERENCES

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